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PLASTIC BUCKLING OF AXIALLY LOADED ALUMINIUM CYLINDERS: A NEW DESIGN APPROACH

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ABSTRACT

The results of a wide F.E.M. analysis on the imperfection sensitivity of axially loaded aluminium cylinders are used to investigate buckling modes occurring in case of relatively thick cylinders (R/t < 200 ÷ 250). The combined effect of geometrical imperfections, inelastic behaviour of material and boundary conditions is considered in order to set-up a refinement of rules given in prEN1993-1-6 dealing with steel shells. The proposal allows for a further exploitation of the cylinder buckling strength in plastic range, which is why it seems rather suited to applications in the field of civil engineering. To this purpose, a special requirement on the initial allowable imperfection level is defined, corresponding to a quality class higher than EC3 class A. Because of its features, the proposal presented herein could be profitably used for the new Part 1-5 "Supplementary rules for shell structures" of Eurocode 9 (prEN1999-1-5), presently under development, which is the very first codification issue at European level dealing with aluminium shell structures.

1. INTRODUCTION

Two main problems have traditionally characterised and troubled both research and codification on shell structures. On one hand, the actual difficulty to face the strong imperfection sensitivity of shells by means of simple design rules suitable for codification purposes; on the other hand, the problem to take into proper account the effect of the inelastic behaviour of material on both buckling behaviour and ultimate load. As a consequence of this, the state of codification appeared for long time rather incoherent, with a rather unsatisfactory treatment of the afore mentioned aspects. As far as the effect of imperfection is concerned, significant advances have been recorded in the latest years, due to the provisions given in the part of Eurocode 3 dealing with steel shells (prEN1993-1-6) [1,2]. In this code, the actual imperfection magnitude is allowed for by means of suitable quality classes defined according to the initial imperfection level of the shell. Within each class, the effect of imperfection is evaluated through the traditional "*Lower Bound Design Philosophy*", according to which a knock-down factor of buckling loads, denoted by α, is fitted as lower limit of the scattered experimental data. Nevertheless, this dependence is mostly related to the imperfection magnitude rather than to its actual distribution alongside the shell surface. Referring to the material inelastic behaviour, its effect is taken into account in the code by means of a regression curve, which does not permit to distinguish between materials with different inelastic features. Also, the particular feature of plastic buckling, namely the axisymmetric "*elephant foot*" buckling mode, is not allowed for in the code, despite its different (lower) imperfection sensitivity compared with the elastic "*diamond shaped*" elastic buckling mode. This aspect is particularly important for round-house-type materials, namely stainless steels and aluminium alloys, whose behaviour is peculiarly hardening and is characterised by significant inelastic deformations before the conventional yield point.

Based on these considerations, this paper wants to represent an attempt to refine the EC3 imperfection sensitive approach, in order to allow for the specific features of plastic buckling. First of all, the conditions for the onset of plastic buckling are investigated by means of non linear F.E.M. analysis, carried out with the ABAQUS code. To this purpose, a wide parametric analysis on aluminium alloy circular cylinders under axial loaded has been performed, aimed at highlighting the quite peculiar imperfections sensitivity of such structures when plastic buckling occurs. Then, the numerical results have been evaluated in the view of codification, with the purpose to define a possible approach to buckling curves in which due account of precritical strain is taken. This led to the introduction of a further imperfection class, placed above EC3-class A, characterised by purely plastic buckling and, hence, by a reduced imperfection sensitivity compared with elastic buckling. As this would mostly involve relatively thick cylinders, it is thought that it could be of particular interest in the field of civil applications. Such proposal could be profitably used for the prEN1999-1-5 "Supplementary rules for shell structures" of Eurocode 9, which is presently under development within the activity of the CEN/TC250-SC9 Committee (Chairman F.M. Mazzolani). This new part of EC9, modelled along the existing prEN1993-1-6 dealing with steel shells, is the very first codification issue at European level dealing with aluminium shell structures and, thus, could profit of the results achieved in this research.

2. BEHAVIOURAL ASPECTS OF SHELL PLASTIC BUCKLING

As well known, the most important aspect of the buckling behaviour of axially loaded cylinders is their high imperfection sensitivity, due to the asymmetric stable-unstable response at bifurcation point (Figure 1a). This can involve a more or less great reduction of the actual buckling loads compared with the theoretical predictions, depending on the R/t ratio as well as on both imperfection magnitude and distribution. Most of the existing data confirm that the greatest imperfection sensitivity is typical of thin cylinders ($R/t > 300 \div 500$), which fail in purely elastic range. Nevertheless, both theoretical and numerical investigations show that imperfections can play an important role also in case of thicker shells ($R/t < 200$), in which the interaction between imperfections and plasticity effect can occur, with a strong influence on both buckling modes and postcritical behaviour.

The buckling behaviour of axially loaded cylinders is characterised by a twofold feature. In case of elastic buckling, the structure generally fails at a load much below the bifurcation load, giving place to the typical asymmetric "*diamond shaped*" postcritical pattern (Figure 1b), with predominant inward buckles, mostly located in the intermediate section of the cylinder; in this case, an as little as negligible influence of the boundary condition is usually observed.

When significant plastic deformations arise before buckling, namely in thicker shells, the deflected shape at buckling is completely different, consisting in one or two outward axisymmetric folds placed close to the cylinder loaded ends (Figure 1c). In this case the buckling behaviour is deeply influenced by the boundary conditions, as well as by the plastic deformations arising after buckling, which strongly reduce the load bearing capacity. In this case, the shell imperfection sensitivity is much different, being affected by the plastic flow in material rather than by the geometrical non linear effect of surface imperfections.

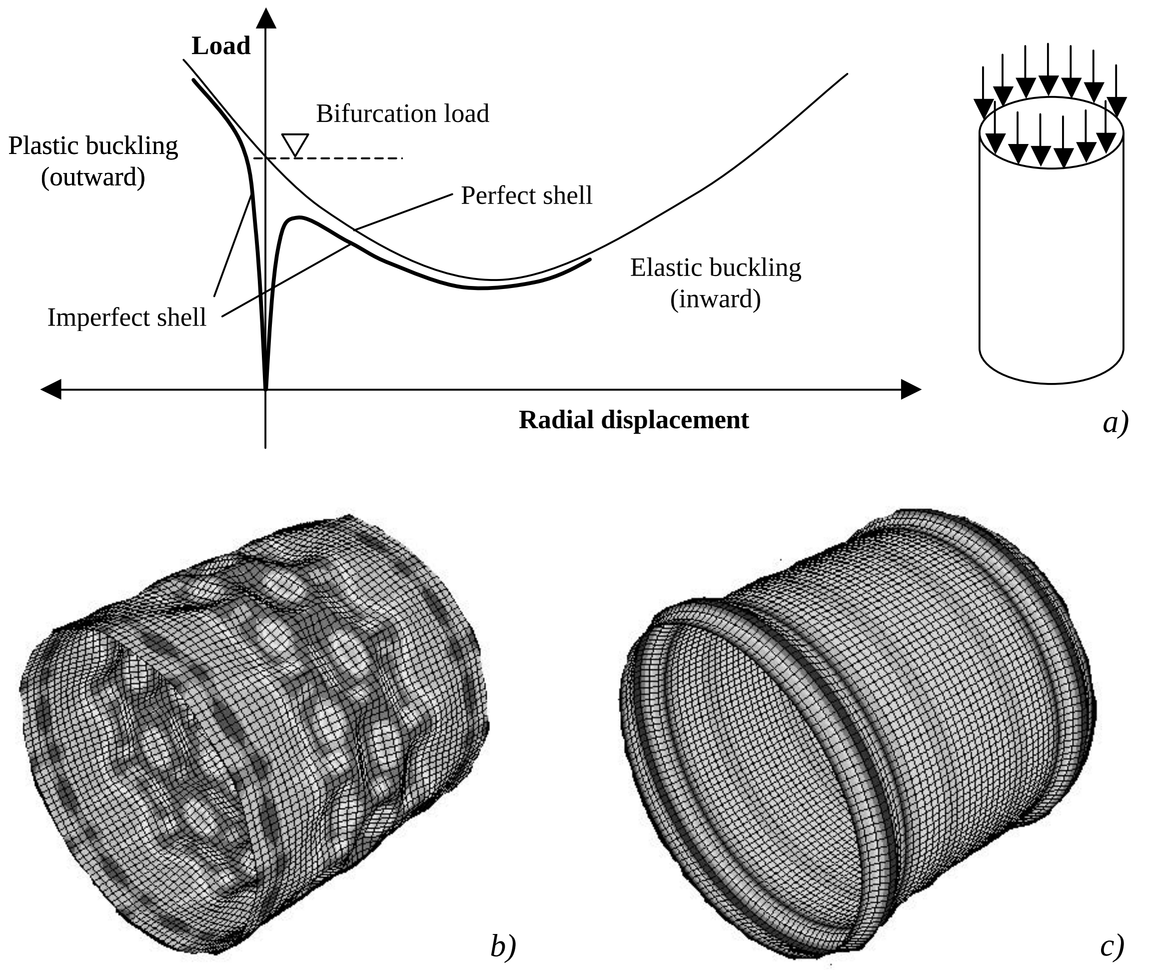

Figure 1: The basic critical and postcritical equilibrium paths of axially loaded cylinders (a); elastic buckling pattern (b); plastic buckling pattern (c).

Relatively few attempts to investigate plastic buckling exist in technical literature on shells. The majority of them is concerned with experimental tests without analysing the influence of imperfections on the buckling mode. An exhaustive theoretical study on the effect of an axisymmetric imperfection on inelastic buckling of axially compressed cylinders was made by Gellin in 1979 [3]. It may be considered as an extension to the inelastic case of the well known results obtained by Koiter [4] for the elastic case. Nevertheless, contrary to the experimental observations, the postcritical response calculated by Gellin is always asymmetric ("*diamond shaped*"). On the other hand, according to the bifurcation approach followed by Gerard (1962) [5], axisymmetric buckling always occurs in an imperfection-free cylinder when any small precritical plastic deformation arises in the shell wall. Nevertheless, both experimental and numerical investigation show that, even though an axisymmetric instability is predicted theoretically, the nature and magnitude of imperfections can play a significant role in activating postbuckling paths which are different from the expected ones. In other words, the shell postbuckling response can be even asymmetric if, for example, the initial imperfection is directed according to elastic asymmetric critical mode, as well as if it contains some initial inward deflections. At the same time, it should be considered that, for comparatively small imperfections, the predicted fully axisymmetric buckling would occur, regardless of the actual imperfection distribution. In such a case, the little imperfection magnitude allows for comparatively greater precritical plastic deformations, which make easier the onset of axisymmetric buckling. All these observations lead to believe that, for some types of initial imperfections, a limit value of imperfection exists beyond which the postbuckling pattern turns to asymmetric.

From the technical point of view, it is important to calculate this limit as a function of both geometry and material properties, being both buckling and postbuckling strength strongly affected by the critical mode. A recent development in the study of imperfection sensitivity of relatively stocky cylinders in compression has been made by Mandara [6], Mandara & Mazzolani [7] and Mazzolani et al. [8,9,10,11] by means of F.E.M. simulation. The analysis presented herein, similarly to those shown in [8,9,10,11], is based on the non linear ABAQUS F.E.M. Code [12] to highlight the influence of both plasticity and imperfections on the ultimate load and postcritical behaviour of stocky aluminium alloy cylindrical shells subjected to axial compression.

3. DESCRIPTION OF THE PARAMETRIC ANALYSIS

3.1. General

R/t and *L/R* values have been chosen in order to cover cylinders characterised by elasto-plastic buckling, namely R/t = 50, 100, 200 and L/R = 2. A typical F.E.M. mesh of the structural model is visible in Figure 1b,c. Three types of alloys have been considered (Table 1), chosen according to the distinction between Hardening alloys and Heat-treated alloys, already considered in the European Recommendations on Aluminium Alloy Structures, and also introduced in the ENV1999-1-1 with slight modifications (Strong hardening and Weak hardening alloys). The exponent $n_{R.O.}$ of the Ramberg-Osgood law has been evaluated by assuming $n_{R.O.} = f_{0.2}/10$, with $f_{0.2}$ expressed in N/mm^2 (Mazzolani [13]). A further group of alloys with higher mechanical features ($f_{0.2}$ = 300N/mm^2 and $n_{R.O.}$ = 30) has been considered in order to cover high strength alloys (e.g. the 6000 and 7000 alloy series).

A static analysis of the unstable structural behaviour has been performed by means of F.E.M. simulation. In this case the limit load of the imperfect structure is evaluated together with the postcritical response as a function of the initial imperfection. Such a kind of approach is suitable when no snap-through or mode jumping is expected, as usually happens in the case of relatively thick cylinders which buckle in plastic range. It is worthy to be observed that in case of very thin shells, which collapse in elastic range, snap-through and mode jumping are liable to occur. In this case, due to the sudden changes in the postcritical equilibrium configurations, the static approach could be inaccurate due to the strong influence of both inertia forces and load application procedure. For this reason, a fully non linear dynamic analysis of the cylinder should be performed in this case. When thick cylinders are faced, the difference as respect to a more complex dynamic approach is unessential up to the limit load is reached, the evaluation of the ultimate load bearing capacity being possible with a more cost effective static analysis. Moreover, the use of a quasi-static procedure enables to make the buckling response independent of the possible testing procedures, by highlighting the postbuckling equilibrium paths.

The F.E.M. analysis has been carried out by means of the ABAQUS code using the RIKS method, for the solution algorithm, the *DEFORMATION PLASTICITY option for material law and four-node, reduced integration S4R5 shell elements. The ABAQUS option *DEFORMATION PLASTICITY has been adopted, as it is slightly more conservative than the option *PLASTIC. One could observe that the use of *DEFORMATION PLASTICITY option can lead to a scarcely accurate evaluation of the postcritical behaviour because it does not take into account the elastic unloading arising after buckling. On the other hand, in many circumstances it has been observed that in the close vicinity of the buckling point the structural response is essentially governed by the tangent modulus of material and this corresponds to the absence of elastic unloading during buckling. Moreover, the hypothesis of full loading during buckling has many times proved to supply more realistic evaluation of collapse loads, both for beams (e.g. the *Shanley*'s column) and for bidimensional structures, such as plates and shells. In addition, the *DEFORMATION PLASTICITY option is based on the pluriaxial formulation of the classical Ramberg-Osgood law, and this makes it particularly suitable for carrying out a parametric analysis on aluminium alloys shells.

	$f_{0.2}$ (N/mm^2)	$n_{R.O.}$
Strong hardening alloys	100	10
Weak hardening alloys (Heat-treated alloys)	200	20
	300	30

Table 1: Aluminium alloys considered in the analysis

3.2. Modelling of structural imperfection

The definition of structural imperfection is one of the basic concerns of the research on shell stability. For very thin shells initial imperfections can be evaluated by means of direct measurement on full scale specimens and then interpreted by means of a multiple *Fourier* series. The analysis of the imperfect structure can be subsequently performed through F.E.M. non linear calculation. Nevertheless, this procedure, followed by many researchers for investigating the stability of thin shells, requires a thorough imperfection measurement, which is usually rather expensive. Furthermore, it provides an effective description of initial imperfection for a given manufacturing class of shell only, characterized by a well defined imperfection distribution. For this reason the results of stability analysis are valid for that class of shell only, and can not be generalized. In the present analysis a different approach has been followed, assuming the following imperfection model:

$$w = w_0 \sum e^{-k_{1x}(x-x_o)^2} \cos\left[k_{2x}p \frac{(x-x_o)}{L}\right] e^{-k_{1y}(y-y_o)^2} \cos\left[k_{2y} \frac{(y-y_o)}{R}\right] \tag{1}$$

This model is suitable for interpreting the imperfection affecting the relatively thick shells, being these imperfections generally smoother as compared to those potentially present in thin shells. By assigning suitable values to k_{1x}, k_{1y}, k_{2x}, k_{2y}, x_0 e y_0, Equation (1) is able to describe imperfection distributions similar to both axisymmetric and asymmetric critical and postcritical modes, as well as any combination of them. As a rule, initial imperfection distributions similar to the critical modes have been assumed in the analysis. This corresponds to give the parameters k_{2x} and k_{2y} a value corresponding to the number of longitudinal (m) and circumferential (n) waves, respectively. In this way the most severe condition for the buckling response has been investigated, so that a lower bound of the ultimate load carrying capacity has been determined as a function of the magnitude of the initial imperfection. Moreover, in spite of the purely theoretical meaning of the assumed imperfection pattern, the obtained results have shown to be sufficiently general to put into evidence the interaction between plasticity and imperfection effects. Figure 2 shows some of the initial imperfect configurations considered in the analysis, obtained by assuming the parameters corresponding to either elastic or plastic critical mode, including random combinations of them.

In order to assume imperfection patterns distributed according to shell critical modes, the critical bifurcation modes and relevant critical loads have been evaluated both for elastic and plastic instability. Effects of plasticity on the buckling load have been taken into account by means of a suitable plasticity reduction factor η, obtained from literature. A synopsis view of bifurcation load, critical modes and plasticity factor is given in Table 2.

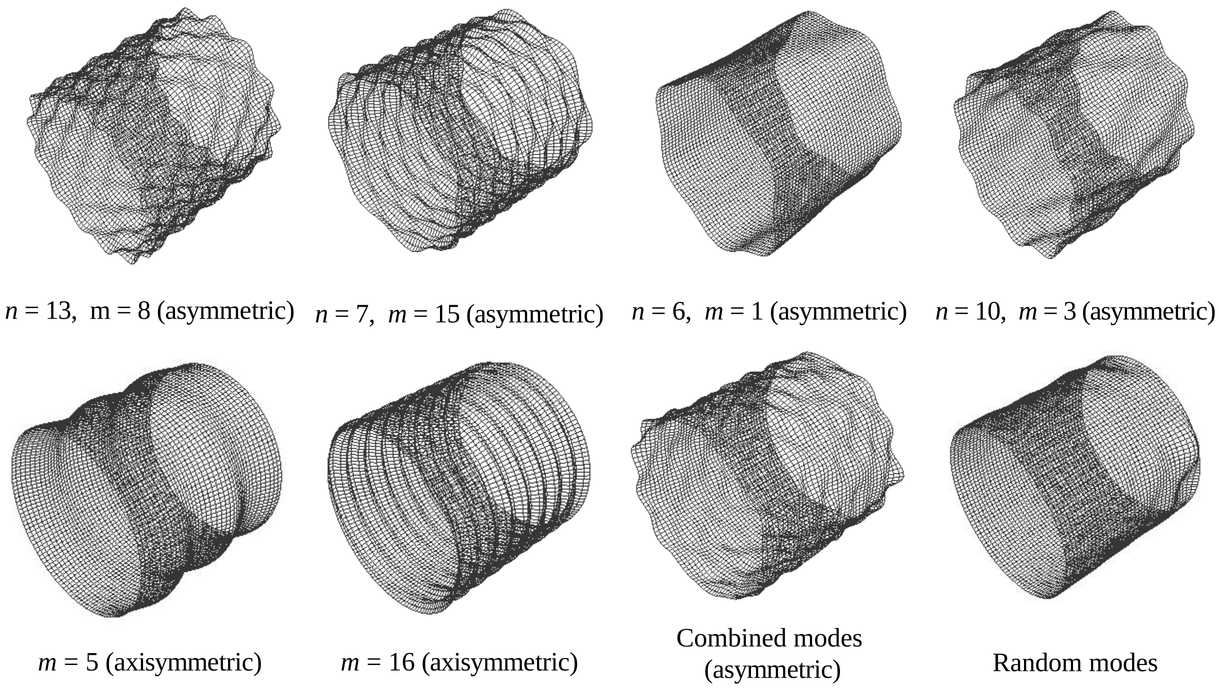

Figure 2: Some of imperfection distributions considered in the analysis

In case of axially loaded cylinders which buckle in purely elastic range, the equation (see Table 2):

$$\frac{(m^2 + (nL/\pi R)^2)^2}{m^2} = \frac{2}{\pi}\frac{L^2}{Rt}\sqrt{3(1-\nu^2)} \qquad (2)$$

shows that many critical modes are possible for the same value of the buckling load. These modes can be individuated by calculating the minimum value of the bifurcation load for couples of integer values of *m* and *n*. In order to make the selection of *m* and *n* easier, Equation (2) can be plotted in the *m,n* plane where, by means of an appropriate choice of axis scale, it represents the equation of a circle ("*Koiter*" circle). In case of plastic buckling, the critical axisymmetric mode derived by a bifurcation analysis has been considered.

The minimum critical load in the plastic range for the perfect shell can be expressed in the general form:

$$\sigma_{cr,p} = \eta\,\sigma_{cr,el} \qquad (3)$$

where η is a reduction factor introduced to take into account the plasticity effect. Values obtained by *Gerard* [5] have been indicated in Table 2, evaluated under the assumption of incompressible material in the plastic range and following the plastic deformation approach.

3.3. Discussion of results

The F.E.M. imperfection sensitivity analysis has emphasised the strong influence of the initial imperfection pattern on the ultimate load P_u, when directed according to the critical modes [8]. For the sake of comparison, the theoretical elastoplastic critical load $P_{cr,th} = 2\pi Rt\sigma_{cr,p}$ has been also evaluated, calculating E_t and E_s according to *Ramberg-Osgood* law. In general, for $w_0/t = 0$, bifurcation loads evaluated by means of the *Gerard*'s η coefficient are in good agreement with F.E.M. prediction, especially in case of clamped ends. Figure 3 shows some cases of imperfection sensitivity for imperfection directed according the asymmetric elastic "*chessboard*" mode (Imperfection 1, 2, 3 and 4), which resulted to be more dangerous compared to axisymmetric imperfection modes. Limits corresponding to EC3 quality classes A, B and C are also plotted.

Elastic bifurcation load	Number of circumferential (n) and axial (m) waves at elastic buckling	Plasticity reduction factor	Number of axial (m) waves at plastic buckling
$s_{cr,el} = \frac{E}{\sqrt{3(1-v^2)}}\frac{t}{R}$	$\frac{(m^2+(nL/pR)^2)^2}{m^2} = \frac{2}{p}\frac{L^2}{Rt}\sqrt{3(1-n^2)}$	$h = \frac{\sqrt{E_t E_s}}{E}$	$m = \frac{L}{p}\sqrt{12/Rt\sqrt{6+9\frac{E_t}{E_s}+\frac{E}{E_t}}}$

Table 2: Critical load, plasticity reduction factor and wave number at both elastic and plastic buckling for axially loaded cylinders

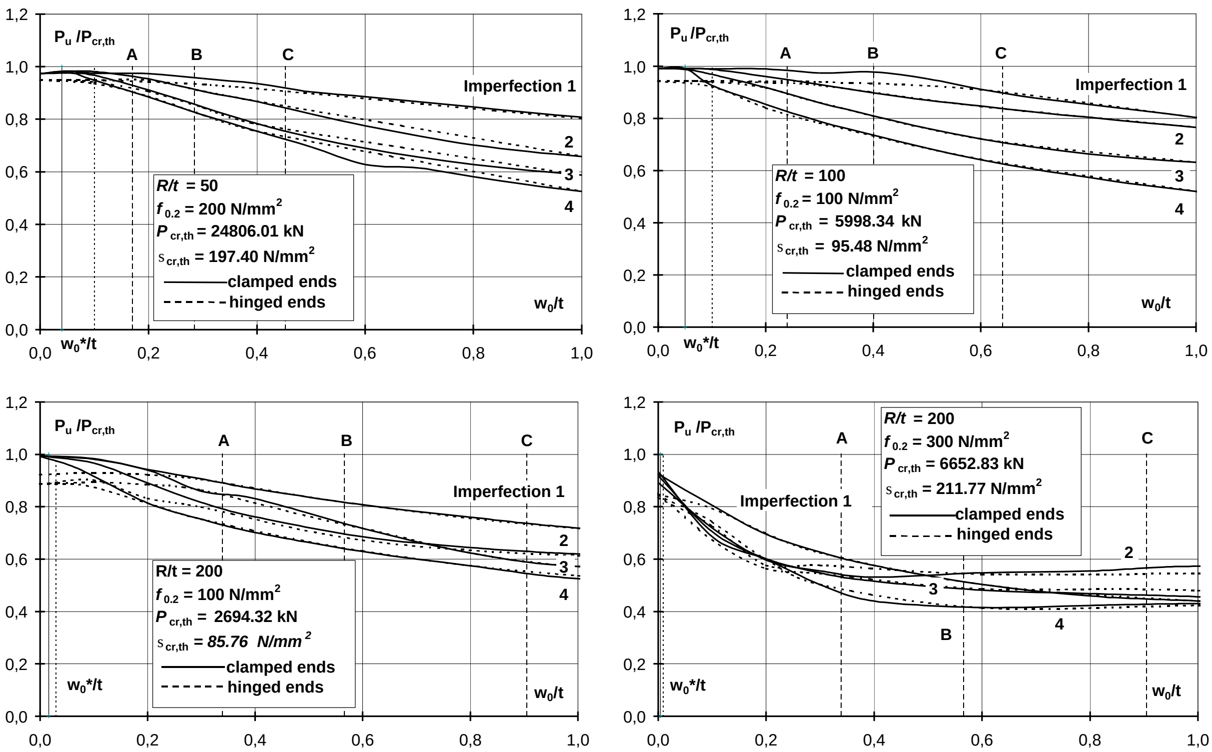

Figure 3: Typical imperfection sensitivity curves for some of cases allowed for (asymmetric imperfection)

The influence of the boundary conditions deeply depends on the imperfection type and, as a consequence, on the buckling character. In particular, it is rather significant when the buckling is axisymmetric, whereas tends to disappear for asymmetric buckling (Figure 3). This can be easily explained by observing that in the axysimmetric instability the annular folds are placed close to the restrained ends and, for this reason, the buckling strength is strongly affected by the boundary conditions. On the contrary, in the asymmetric instability the buckling of shell surface starts as a rule at cylinder midlength and is scarcely influenced by the end restraints.

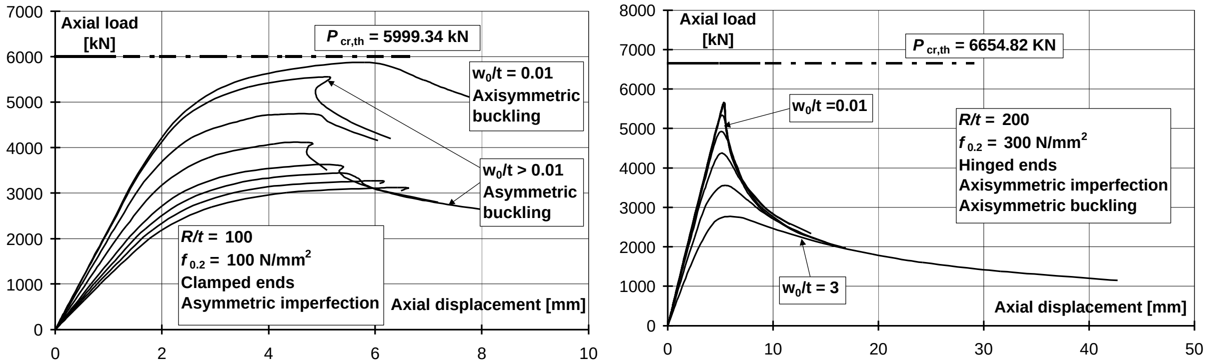

Figure 4: Typical load-displacement curves for cylinders failing in plastic range: asymmetric imperfection (left), axisymmetric imperfection (right)

As far as the asymmetric imperfection is concerned, it can be observed that the typical snap-through behaviour of perfectly elastic shell tends to disappear when the plasticity effects are taken into consideration, at least for small imperfection magnitudes (Figure 4, left). In all the examined cases, the transition from the asymmetric to axisymmetric buckling corresponds to a critical value of imperfection w_0*/t. Around this imperfection value, both types of buckling can occur simultaneously. It is also possible to observe a great difference in postcritical behaviour when going from the axisymmetric to asymmetric diamond shaped buckling, being the former rather progressive and imperfection insensitive, whereas the latter involves a sharp drop in load carrying capacity after buckling.

When considering axisymmetric imperfection, buckling is always symmetric with the ultimate load decreasing with the imperfection magnitude. The drop in load postcritical capacity is as much noticeable as long as the imperfection level decreases. As in the case of asymmetric imperfection, the residual structural strength tends to uniform in the fully developed postcritical range, regardless of the initial imperfection magnitude (Figure 4, right).

In the case of imperfections different from those corresponding to the critical modes, the buckling behaviour is rather different. In general, imperfection sensitivity for concentrated or linear defects is remarkably lower than for imperfection distributions similar to critical modes. Also, the influence of the boundary conditions tends to disappear when the buckling deflection is predominantly developed in the intermediate region of shell surface. For more details see [8].

Eventually, a great scatter of results can be observed, which confirms the commonly acknowledged experimental performance of axially loaded cylinders (Figure 3). Such scatter increases as long as R/t ratio and elastic limit $f_{0.2}$ increase. The lower bound of numerical data corresponds, with good approximation, to imperfection distributions directed according to critical modes taken from the *Koiter* circle. Figure 3 shows some of such imperfection sensitivity curves, together with a conservative evaluation of the limit imperfection w_0*/t. It is important to underline that, below the limit imperfection value w_0* the buckling is always axisymmetric, regardless of the actual imperfection distribution. This is a peculiar feature of axially loaded cylinders failing in plastic range, which can be exploited for the set-up of a check criterion, as shown in the next paragraph. In particular, it is possible to allow for the fact that the shell exhibits a comparatively small imperfection sensitivity when $w_0 < w_0*$. On the other hand, a significant influence of the end restraint conditions is observed, which has to be taken into due account. Based on the results of F.E.M. simulation, a simplified expression for w_0^*/t, valid as long as $R/t \le a/b$, can be put in the form (Figure 5):

$$\frac{w_0^*}{t} = \left(a - b\frac{R}{t}\right)\left(\frac{200}{f_{0.2}}\right) \tag{4}$$

where coefficients *a* and *b* depend on end restraint conditions of the shell.

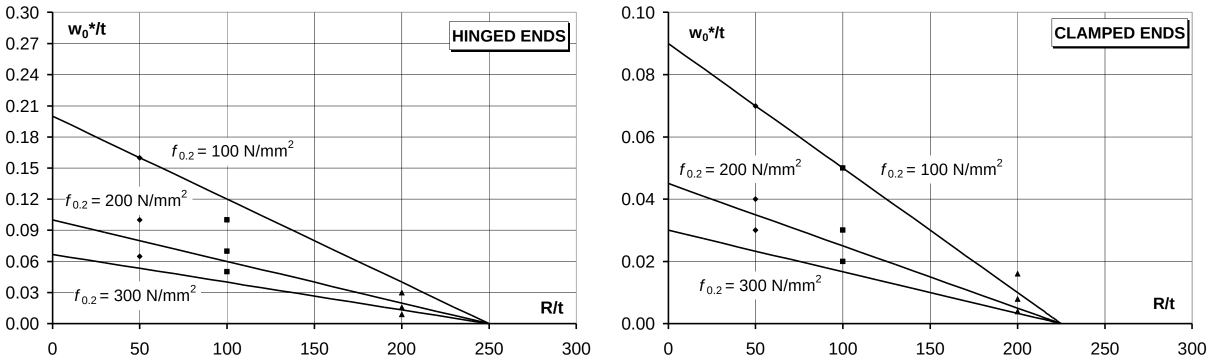

Figure 5: Values of the limit imperfection w_0^*/t

4. PROPOSAL FOR CODIFICATION

Due to the peculiar aspect of plastic buckling, a special procedure for checking axially loaded cylinders against buckling is developed herein. The proposed method applies to the cases when some inelastic deformations are attained before buckling, as happens as a rule in relatively thick cylinders made of hardening materials. In addition, for the procedure to be applied, the imperfection level should not exceed the limit value w_0^*/t given in the previous paragraph. If the existing criterion given in the prEN1993-1-6 for steel shells is assumed, this leads to the definition of a supplementary quality class, placed above the upper class defined in EC3, namely the quality class A. Quality classes in EC3 are defined according to fabrication tolerance parameters like the cylinder out-of-roundness parameter U_r, the accidental eccentricity parameter U_e and the so-called dimple tolerance parameter $U_{0,max}$, which is directly related to the surface geometrical imperfection w_0 by means of the position $w_0/\ell_{gx} = U_{0,max}$, where $\ell_{gx} = 4(Rt)^{0.5}$ is the gauge length assumed in the code for measuring geometrical imperfection. From the previous position it is possible to relate the nondimensional imperfection w_0/t to $U_{0,max}$ in the following way:

$$\frac{w_0}{t} \le \frac{U_{0,\max}\ell_{gx}}{t} = 4U_{0,\max}\sqrt{\frac{R}{t}} \tag{5}$$

Depending on the values reached by the above parameters, it is possible to locate a given cylinder in one of the three quality classes A, B, or C allowed for in the code. For each quality class a quality parameter Q is given in EC3, to be used in the calculation of the imperfection reduction factor α_x. Limit values of $U_{0,max}$ as provided in EC3 are given in Table 3, where the expressions relevant to the proposed A-plus Class are also shown. Such relationships are derived directly by Equation (4) considering both the position $\ell_{gx} = 4(Rt)^{0.5}$ and Equation (5). The corresponding values of the quality parameter Q are given in Table 4, together with the formula for the calculation of the imperfection reduction factor α_x, in which $\lambda_x = (f_{0.2}/\sigma_{cr,e})^{0.5}$ and $\lambda_{x,0} = 0.1$ or 0.2 depending on the alloy. This formula is slightly different compared to the one supplied in EC3, in order to better fit buckling curves in plastic range [11].

Fabrication tolerance quality class	Description	Value of $U_{0,.max}$ ($f_{0.2}$ in N/mm^2)	
		Clamped ends	Hinged ends
Class A-plus	Excellent	$\frac{1}{f_{0.2}}\left(2.25\sqrt{\frac{t}{R}}+0.01\sqrt{\frac{R}{t}}\right)$	$\frac{1}{f_{0.2}}\left(5\sqrt{\frac{t}{R}}+0.02\sqrt{\frac{R}{t}}\right)$
Class A	Very high	0,006	
Class B	High	0,01	
Class C	Normal	0,016	

Table 3: Formulas of the dimple tolerance parameter $U_{0,\max}$ for the proposed A-plus Class together with values given in EN1993-1-6 for quality classes A, B and C

Fabrication tolerance quality class	Description	Q		α_x
		Clamped ends	Hinged ends	
Class A-plus	Excellent	60	50	$a_x = \dfrac{1}{1+2.60\left(\frac{1}{Q}\sqrt{\frac{0.6E}{f_{0,2}}}\left(l_x - l_{x,0}\right)\right)^{1.44}}$
Class A	Very high	40		
Class B	High	25		
Class C	Normal	16		

Table 4: Values of the quality factor Q and imperfection reduction factor α_x for the quality classes allowed for in the new proposal

Values of $U_{0,\max}$ in Table 3 for A-plus Class correspond to cylinders which always buckle in plastic range, that is with a axisymmetric pattern. Of course, both the out-of-roundness U_r and the accidental eccentricity parameter U_e should be properly modified in order to fit the quality Class A-plus. In order to predetermine whether a given Class A-plus cylinder will buckle in plastic range or not, it may be conventionally assumed that must result $\eta \leq 0.95$ [14]. For a given alloy, represented by assigned values of $f_{0.2}$ and $n_{R.O.}$, from Equation (3) it is possible to calculate the corresponding *R/t* value to be assumed as the limit for which $\eta \leq 0.95$. This can be easily done by assuming the *Ramberg-Osgood*'s law $e = s/E + 0.002\left(s/f_{0.2}\right)^{n_{R.O.}}$ as material model. The factor α_x given in Table 4 can be taken as imperfection reduction factor, assuming Q = 50 or 60 depending on whether the cylinder ends are hinged or clamped, respectively. The corresponding *R/t* limit values are given in Table 5 as a function of $f_{0.2}$. The squash limit slenderness $\lambda_{x,0}$ has been taken equal to 0.1 for strong hardening alloys ($f_{0.2} \leq 150$ N/mm^2) and to 0.2 for weak hardening alloys ($f_{0.2} \geq 200$ N/mm^2), according to the distinction introduced into EC9. For *R/t* values above those of Table 5, buckling is supposed to be purely elastic and, hence, the shell cannot belong to the Class A-plus. Conversely, for *R/t* values below those of Table 5, buckling may be plastic provided the imperfection level complies with the Class A-plus limits given in Table 3. Otherwise, the cylinder has to be classified as Class A, B or C and checked accordingly.

Suitable buckling curves for Class A-plus cylinders can be plotted according to the proposal presented in [11], based on the format already adopted for the buckling of aluminium members in compression and codified into prEN1999-1-1. Adopting the codified Eurocode symbols, the characteristic buckling strength σ_{xRk} is obtained by multiplying the characteristic limiting strength $f_{0.2}$ by a suitable buckling reduction factor χ, expressed as a function of the relative slenderness of the shell λ_x by means of the relationship $\chi = \alpha_x\chi_{perf}$, in which α_x is the imperfection reduction parameter, given in Table 4, and χ_{perf} is the buckling factor for a perfect shell, given by:

$$\chi_{perf} = 1\Big/\left(\phi + \sqrt{\phi^2 - \lambda_x^2}\right) \qquad \text{with} \qquad \phi = 0.5\left[1 + \alpha_{0,x}\left(\lambda_x - \lambda_{x,0}\right) + \lambda_x^2\right] \tag{6}$$

The parameter $\alpha_{0,x}$ depends on the alloy and $\lambda_{x,0}$ is the squash limit relative slenderness. Values of $\alpha_{0,x}$ and $\lambda_{x,0}$ are given in Table 6 for relevant types of alloy. Proposed buckling curves for Class A-plus cylinders are shown in Figure 6 against the 5% lower bound of F.E.M. results [11]. For the sake of comparison, EC3 Class A curves are also plotted, together with some of the experimental results presented in Mandara & Mazzolani [15], referring to stocky extruded aluminium cylinders with a very low imperfection degree loaded in compression.

Hinged ends (Q = 50)						
$f_{0.2}$ (N/mm^2)	100	150	200	250	300	350
$\sigma_u = \alpha\eta\sigma_{cr,e}$ (N/mm^2)	57.6	105.4	154.4	203.8	253.4	303.1
α	0.611	0.697	0.788	0.825	0.852	0.873
R/t	250 (*)	250 (*)	209	166	138	118

Clamped ends (Q = 60)						
$f_{0.2}$ (N/mm^2)	100	150	200	250	300	350
$\sigma_u = \alpha\eta\sigma_{cr,e}$ (N/mm^2)	57.6	105.4	154.4	203.8	253.4	303.1
α	0.658	0.740	0.822	0.855	0.879	0.896
R/t	225 (*)	225 (*)	218	172	142	121

(*) Values of R/t complying with Equation (4) ($R/t \le a/b$).

Table 5: Limit values of the R/t ratio corresponding to a plasticity factor η = 0.95

Alloy	$\lambda_{x,0}$	$\alpha_{0,x}$
Weak hardening alloys	0.2	0.35
Strong hardening alloys	0.1	0.2

Table 6: Values of $\lambda_{x,0}$ and $\alpha_{0,x}$ for the proposed buckling curves

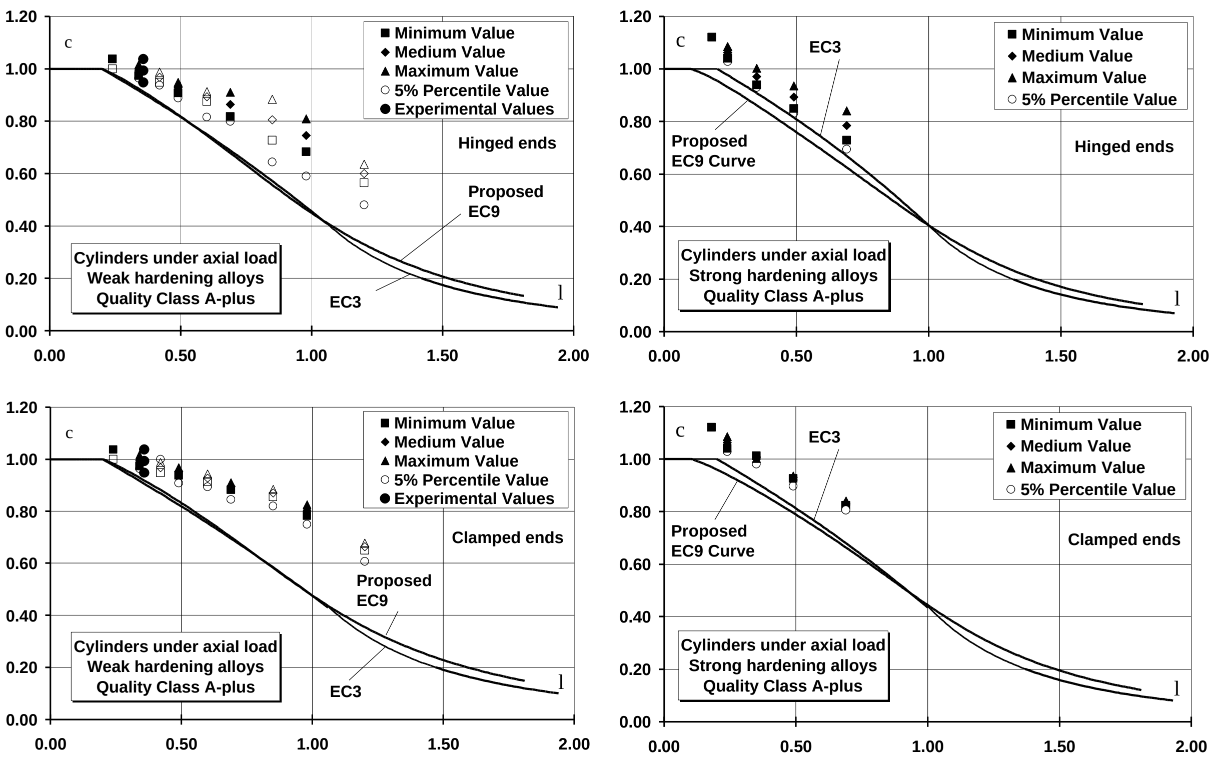

Figure 6: Proposed buckling curves for quality Class A-plus

5. CONCLUSIVE REMARKS

An attempt to refine the EC3 approach to the buckling of axially loaded cylinders has been presented in this paper, in order to take into account the peculiar features of plastic buckling, exploiting in the meantime some benefits involved by its comparatively lower imperfection sensitivity compared to purely elastic buckling. Conditions required for the onset of plastic buckling have been determined by means of an extensive non linear F.E.M. analysis, which put into evidence the quite particular effect of imperfection observed in this case. Numerical results have then been analysed, aiming at defining a design approach to buckling allowing for the influence of precritical strains. Assuming the prEN1993-1-6 shell buckling formulation as a basic frame, a further shell quality class, placed above the EC3 Class A, and its relevant buckling curves have been introduced. As this new class mostly involves relatively thick cylinders failing in plastic range, it could be of particular interest for aluminium shells because of their strain-hardening mechanical behaviour. Also, it would represent an important difference from the codification point of view between Eurocode 9 and Eurocode 3.

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