ICTWS 2004
Fourth International Conference on Thin-Walled Structures
Loughborough, England 22-24 June, 2004

BUCKLING OF ALUMINIUM SHELLS: PROPOSAL FOR EUROPEAN CURVES

F.M. Mazzolani [1], A. Mandara [2] and G. Di Lauro [2]

[1] Department of Structural Analysis and Design, University of Naples Federico II
Naples, Italy
[2] Department of Civil Engineering, Second University of Naples
Aversa (CE), Italy

ABSTRACT

An imperfection sensitive design criterion for aluminium alloy cylinders subjected to axial load, external pressure and torsion is presented in this paper. Buckling curves have been fitted against a comprehensive non linear F.E.M. analysis of cylinder imperfection sensitivity, considering a wide imperfection pattern and accounting for actual inelastic properties of material. Curves shown in this paper are intended to represent a codification proposal for the new Part 1-5 "Supplementary rules for shell structures" of Eurocode 9, which is presently under development within the activity of CEN/TC250-SC9 Committee (chairman F.M. Mazzolani), devoted to the preparation of Eurocode 9 "Design of Aluminium Structures". As this part of EC9 is the very first codification issue at European level dealing with aluminium shell structures, it is hoped that proposed curves could represent a helpful starting point for the general framing of buckling problems.

KEYWORDS

Aluminium Alloys, Shells, Instability, Imperfection Sensitivity, Structural Eurocodes, Buckling Curves.

INTRODUCTION

The new Part 1-5 "Supplementary rules for shell structures" of Eurocode 9 "Design of Aluminium Structures", developed within the activity of CEN/TC250-SC9 Committee (chairman F.M. Mazzolani), is now progressing towards its final stage. This part of EC9 signs an unprecedented milestone in the codification on shell structures, as it is the very first issue at European level dealing with aluminium shells and, therefore, is the first to address specifically the peculiar aspects of such materials when used for shell structures. One of the most relevant problems faced in the preparation of this document has been the set up of buckling curves. Typical features of aluminium in terms of inelastic behaviour, in fact, result in the buckling response and, hence, in the shell imperfection sensitivity to be far different from mild steel. As a consequence, buckling curves usually adopted for steel, can not be used as they are, but require proper modifications in order to be adapted to aluminium shells. The extent of such modifications has been evaluated by means of a suitable F.E.M. simulation analysis, described in more detail in Mazzolani & Mandara, 2003, Mazzolani *et al.* (2003, 2004) and briefly summarised herein. In such analysis, the main aspects of cylinder imperfection sensitivity have been underlined, with particular emphasis to the effect of material plasticity on buckling. The simulation results have then been matched with EC3 curves, namely the ones given in the European Prestandards ENV1993-1-6 "Shell structures" (Rotter, 1998, Schmidt, 2000), clearly showing that they are not suitable to aluminium shells. For this reason a new set of curves is proposed in this paper, conceived in such a way to closely follow the basic formulation of stability problems as dealt with in EC9. The typical issues of EC9, and mainly those concerning the classification of alloys, have been addressed in this proposal. Cases under consideration are the same covered in ENV1993-1-6, namely circular cylinders under axial load, external pressure and torsion/shear. Also, for the sake of homogeneity, the basic layout of ENV1993-1-6, and in particular the concept of quality classes, have been kept.

SINOPSYS OF F.E.M. RESULTS

Because of the peculiar mechanical behaviour of aluminium alloys, the main concern of the F.E.M. analysis was the imperfection sensitivity of cylinders buckling in inelastic range. In this case an interaction between structural imperfections and plasticity effect can occur, with a strong influence on the buckling modes and postcritical behaviour. Also the deflected shape at buckling may be completely different in case of plastic buckling, and this is particularly evident in case of axially loaded cylinders.

In order to cover cylinders failing by elasto-plastic buckling, both radius to thickness R/t and length to radius L/R ratios have been given values ranging between 25 ÷ 200 and 1 ÷ 4, respectively (Mazzolani & Mandara, 2003, Mazzolani *et al.*, 2003, 2004). Three types of alloys, chosen according to the distinction between Hardening alloys and Heat-treated alloys, have been considered in the analysis (Table 1). The three of them have been taken into account for cylinders under axial load, whereas for cylinders under external pressure and torsion the first two have been allowed for, only. The exponent $n_{R.O.}$ of the *Ramberg-Osgood* law has been evaluated according to the *Steinhardt* proposal, i.e. by assuming $n_{R.O.} = f_{0.2}/10$, with $f_{0.2}$ expressed in N/mm^2 (Mazzolani, 1995). A static analysis of the unstable structural behaviour has been performed via F.E.M. simulation, by means of the ABAQUS non-linear code. This allowed to determine both the limit load and the postcritical response of the imperfect structure as a function of the initial imperfection distribution. The ABAQUS F.E.M. analysis has been carried out using four-node reduced integration S4R5 shell elements, the *RIKS solution algorithm and the *DEFORMATION PLASTICITY option for material law (Mandara & Mazzolani, 1993).

TABLE 1
MECHANICAL FEATURES OF ALLOYS UNDER CONSIDERATION

	$f_{0.2}$ [MPa]	$n_{R.O.}$
Strong hardening alloys	100	10
Weak hardening alloys	200	20
(Heat-treated alloys)	300	30

Structural imperfections have been represented by means of the following model:

$$w = w_0 \sum e^{-k_{1x}(x-x_o)^2} \cos\left[k_{2x}\pi \frac{(x-x_o)}{L}\right] e^{-k_{1y}(y-y_o)^2} \cos\left[k_{2y} \frac{(y-y_o)}{R}\right] \quad (1)$$

By assigning suitable values to k_{1x}, k_{1y}, k_{2x}, k_{2y}, x_0 e y_0, Equation (1) is able to describe an imperfection distribution similar to both axisymmetric and asymmetric critical and postcritical modes. The analysis has been carried out assuming an initial imperfection distribution similar to single or multiple critical modes, which corresponds to give the parameters k_{2x} and k_{2y} a value corresponding to the number of longitudinal (m) and circumferential (n) waves at buckling, respectively. In this way the most severe condition for the buckling response has been investigated, so as to determine a lower bound of the ultimate load carrying capacity as a function of the initial imperfection magnitude. Other types of imperfection (say a concentrated dimple placed at cylinder midlength or a continuous longitudinal groove) have been also considered for axially loaded cylinders only. Examples of imperfection distributions according to Equation (1) are illustrated in Figure 1. In order to widen the field of investigated imperfection patterns, the following alternative expression has been also used (Arbocz & Hol, 1991):

$$w = w_0 \sum_{k=1}^{n_1} \sum_{l=1}^{n_2} \sin \frac{k\pi x}{L} \left(C_{kl} \cos \frac{ly}{R} + D_{kl} \sin \frac{ly}{R} \right) \tag{2}$$

where the coefficients A_{kl} and B_{kl} depend on the constructional features of the shell. By using such an imperfection model it is possible to interpret the sharp gradients which are common in the imperfection pattern of very thin shells. In this analysis such imperfection model has been considered for axially loaded cylinders only, with coefficients A_{kl} and B_{kl} assumed to have a random distribution. A more detailed description of the imperfection distributions assumed in the analysis is given in Mazzolani & Mandara, 2003.

In order to assume imperfection patterns distributed according to shell critical modes, the critical bifurcation modes and corresponding critical loads have been preliminary evaluated for both elastic and plastic instability. Effects of plasticity on the buckling load have been taken into account by means of a plasticity reduction factor η. A complete frame of bifurcation loads, critical modes and plasticity factors is given in Mazzolani & Mandara (2003), Mazzolani *et al.* (2003, 2004) for all relevant load cases considered in the analysis. The minimum theoretical critical load $P_{cr,th}$ in elastoplastic range is expressed in the general form:

$$P_{cr,th} = \eta P_{cr,el} \tag{3}$$

where $P_{cr,el}$ is the purely elastic bifurcation load and η is the reduction factor introduced to take into account the plasticity effect, depending on material tangent $E_t = d\sigma/d\varepsilon$ and secant moduli $E_s = \sigma/\varepsilon$. Values obtained by Gerard (1962) have been assumed for η, evaluated assuming incompressible material in the plastic range and following the plastic deformation approach.

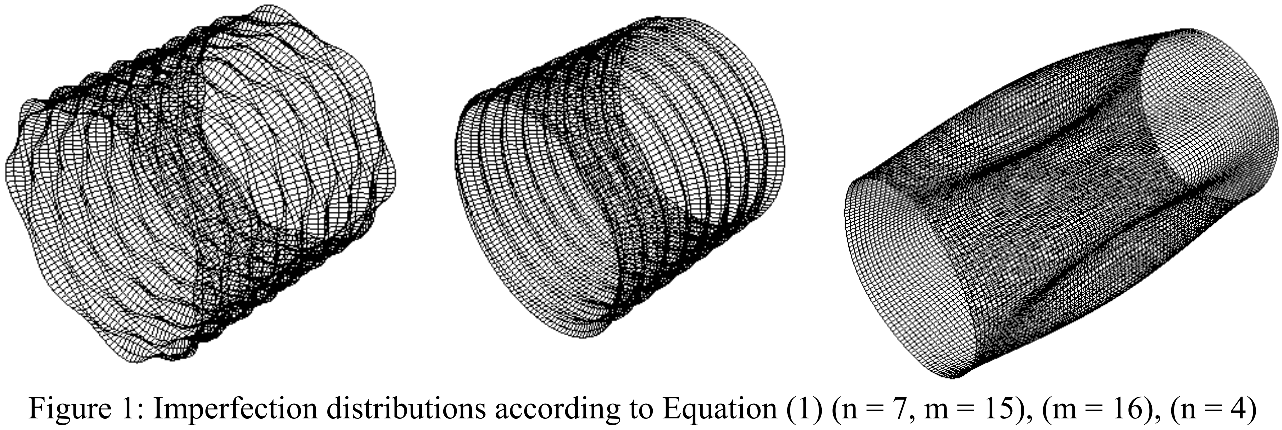


Figure 1: Imperfection distributions according to Equation (1) (n = 7, m = 15), (m = 16), (n = 4)

The F.E.M. analysis has emphasised the strong influence of the initial imperfection pattern, in particular when directed according to the critical modes. Typical imperfection sensitivity curves are plotted in Figure 2, showing the great reduction of the collapse load P_u with the imperfection magnitude w_0, compared with the theoretical elastoplastic critical load $P_{cr,th}$. Limits corresponding to quality classes A, B and C, as defined in ENV1993-1-6, are also drawn. In general, elasto-plastic bifurcation loads according to Gerard's η coefficients are in good agreement with F.E.M. prediction. As a rule, values provided by simulation for $w_0 = 0$ fall on the conservative side with a slight discrepancy, decreasing as long as both the R/t ratio and the elastic limit strength $f_{0.2}$ decrease. In general, it can be confirmed that the effect of initial imperfection increases as long *R/t* and *L/R* ratios, as well as the elastic limit strength $f_{0.2}$ increase, while in the same conditions the effect of plasticity is relatively smaller. Some buckling deflected shapes are shown in Figure 3. Both theoretical predictions and experimental evidences on the buckling pattern are fully confirmed by the F.E.M. analysis for all load cases taken into consideration.

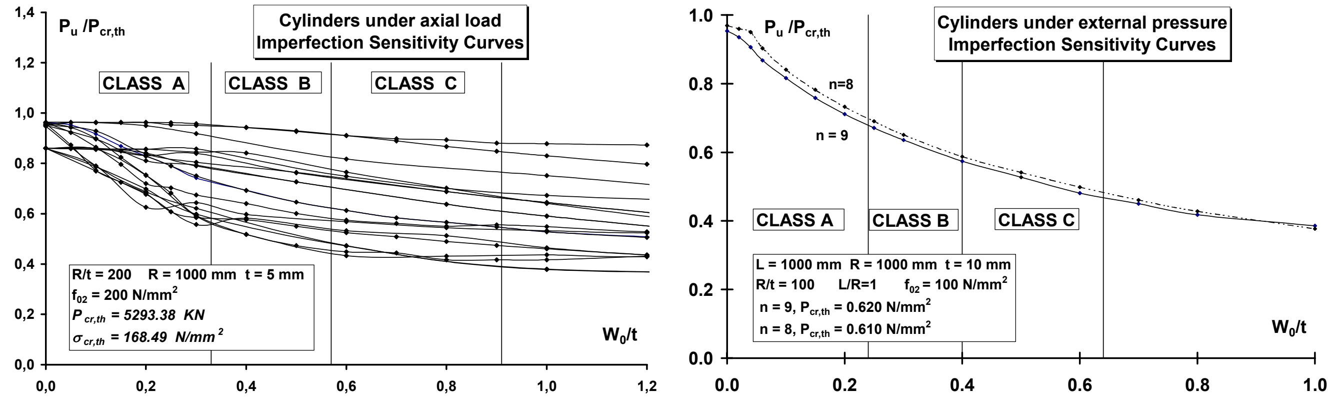


Figure 2 : Typical imperfection sensitivity curves for cylinders (axial load and external pressure)

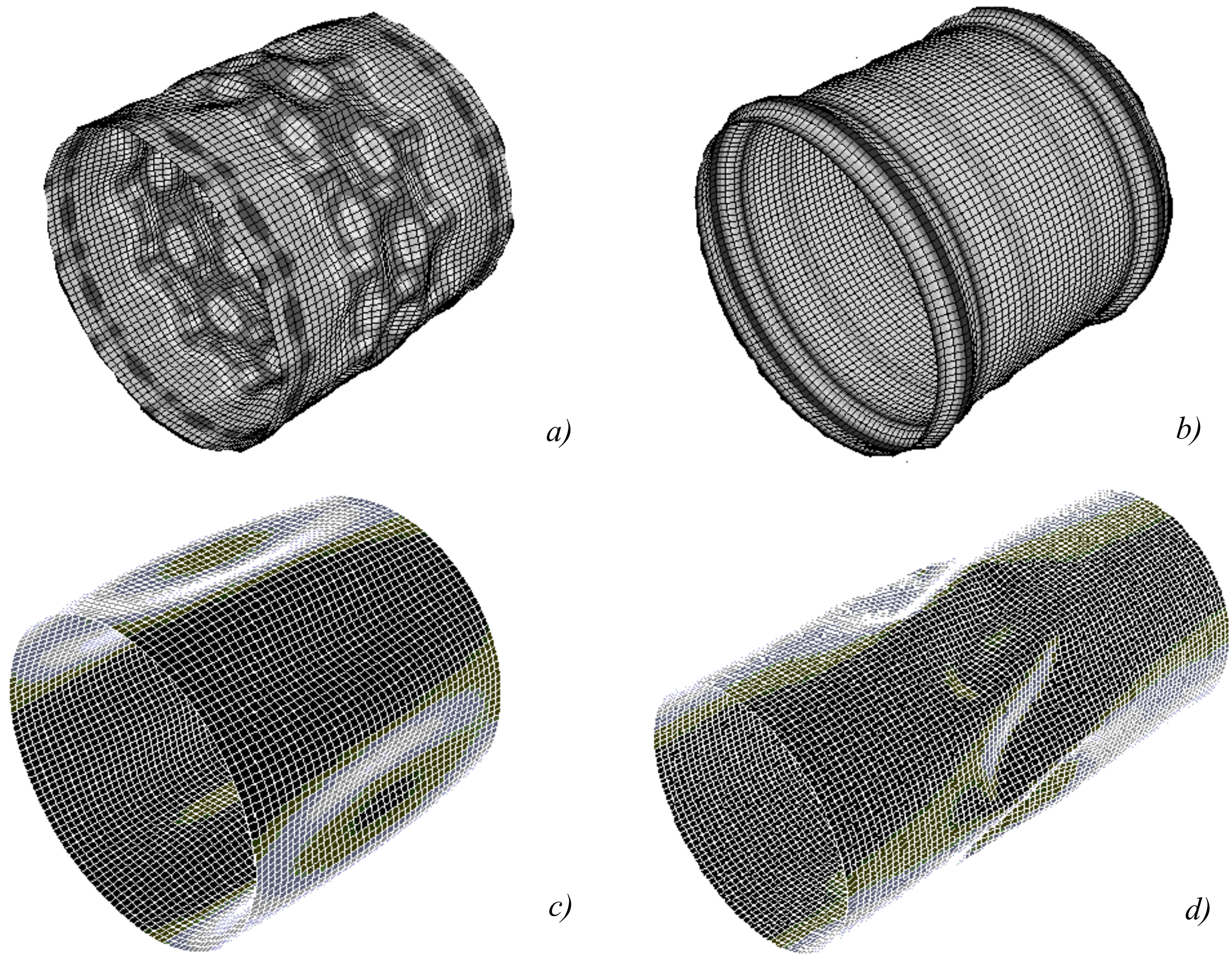


Figure 3 : Characteristic buckling deflected shapes: a) axial load, elastic buckling; b) axial load, plastic buckling; c) uniform external pressure; d) torsion/shear

SEMI-PROBABILISTIC APPROACH TO NUMERICAL DATA

Because of the great scattering observed in numerical buckling data, a further semi-probabilistic analysis has been carried out in order to define a reliable lower bound of buckling curves to be proposed for codification. To this purpose, the whole of numerical data referring to axially loaded cylinders has been treated in stochastic way, that is by assuming the obtained results to be interpreted by a specific probabilistic distribution. Such assumption allows the extrapolation of lower values of ultimate load, corresponding to a given fractile value (e.g. 5%). In this way, a characteristic lower bound can be defined for fitting buckling curves. The *Weibull* distribution has been used to this aim, whose cumulative curve is described by the equation:

$$P(x) = 1 - e^{-(\alpha x)^{\beta}} \tag{4}$$

where α and β are characteristic parameters to be fitted on the basis of available data. From Equation (4) the probability density curve can be obtained:

$$p(x) = \frac{dP(x)}{dx} = \frac{1}{\alpha\beta^{1/\alpha}} x^{(1/\alpha - 1)} e^{-(x/\beta)^{1/\alpha}} \tag{5}$$

The *Weibull* extreme distribution has been already used for the stochastic evaluation of the buckling load of imperfect cylinders (Mendera, 1989), and is well suited to the description of random variables ranging between 0 and 1. Parameters α e β have been estimated on the basis of numerical data divided according to shell quality classes as defined in ENV1993-1-6. A complete frame of α and β values is given in Mazzolani & Mandara (2003), as a function of material, *R/t* ratio and quality class. Some of obtained cumulative curves, in which the stochastic variable *x* has to be assumed as the cylinder buckling load, are shown in Figure 4, together with the corresponding *Weibull* cumulative curves and characteristic 5% lower bound.

PROPOSAL FOR BUCKLING CURVES

A preliminary comparison of simulation results with provisions given in ENV1993-1-6 showed that buckling curves used for steel shells cannot be applied to aluminium shells, because of the strongly different behaviour in the transition region between elastic and plastic range (Mazzolani *et. al.*, 2003). A first attempt to adapt the EC3 approach was proposed by Mazzolani & Mandara (2003), by modifying the λ_0, β and η parameters provided in the piecewise formulation of buckling factor χ given in ENV1993-1-6, that is:

$$\chi = 1,\ \text{for } \lambda < \lambda_0; \qquad \chi = 1 - \beta\left(\frac{\lambda - \lambda_0}{\lambda_p - \lambda_0}\right)^{\eta},\ \text{for } \lambda_0 < \lambda < \lambda_p; \qquad \chi = \alpha/\lambda^2,\ \text{for } \lambda_p < \lambda_0 \tag{6}$$

Nevertheless, this seemed to lead to an unjustified excess of conservativeness and, most of all, to a lack of accuracy in the interpretation of buckling data in the elastic-plastic region. Also, this approach would involve the commonly recognised difference between strong and weak hardening alloys to be completely missing. In order to overcome such limits, an alternative formulation for aluminium shell buckling curves has been presented. It is concerned with the fundamental load cases of axial (meridional) load, external pressure (circumferential compression) and shear (torsion), also considered in the ENV1993-1-6. The proposal is based on the format already adopted for the buckling of aluminium members in compression and codified into EN1999-1-1. Proper account is made for imperfection reduction factors, which are kept equal to those provided into ENV1993-1-6, except for the case of axial compression. The characteristic buckling strengths are obtained by multiplying the characteristic limiting strength $f_{0.2}$ by suitable buckling reduction factors χ, given by:

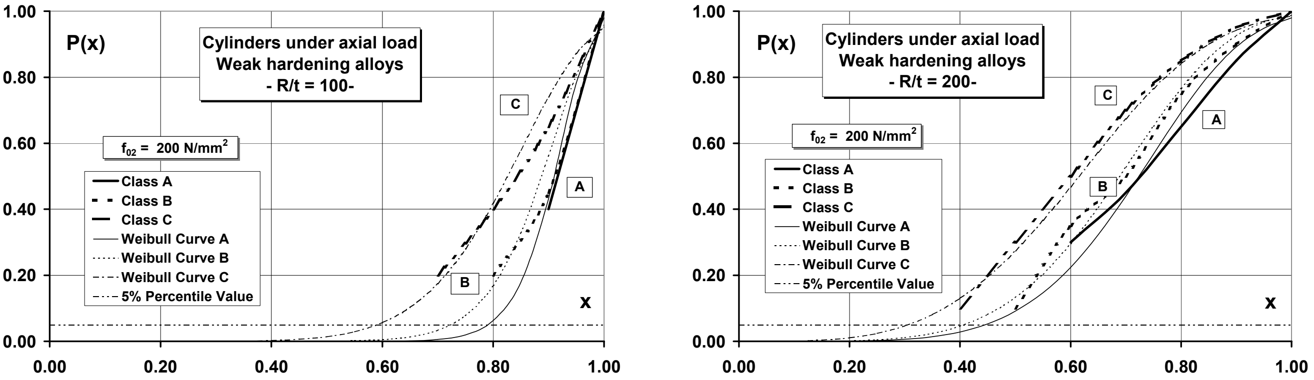


Figure 4 : Semi-probabilistic exploitation of simulation data according to *Weibull* extreme law

TABLE 2

VALUES OF α_0 AND λ_0 AND FOR RELEVANT LOAD CASES AND ALLOY TYPES

Alloy	Axial (meridional) load		External pressure		Shear (torsion)	
	λ_0	α_0	λ_0	α_0	λ_0	α_0
Weak hardening alloys	0.2	0.35	0.3	0.55	0.5	0.3
Strong hardening alloys	0.1	0.2	0.2	0.7	0.4	0.4

TABLE 3

FABRICATION TOLERANCE QUALITY CLASSES AND EXPRESSIONS OF THE IMPERFECTION REDUCTION FACTOR

Fabrication tolerance quality class	Description	Axial (meridional) load		External pressure (α_θ) and shear (torsion) (α_τ)
		Q	α_x	α_θ or α_τ
Class A	Excellent	40	$\alpha_x \frac{1}{1+2.60\left(\frac{1}{Q}\sqrt{\frac{0.6E}{f_{0,k}}}\left(\lambda_x-\lambda_{x,0}\right)\right)^{1.44}}$	0,75
Class B	High	25		0,65
Class C	Normal	16		0,50

$$\sigma_{xRk} = \chi_x f_{0,k}, \quad \sigma_{\theta Rk} = \chi_\theta f_{0,k}, \quad \tau_{x\theta Rk} = \chi_t f_{0,k} / \sqrt{3} \tag{7}$$

where χ_x χ_θ and χ_t refer to axial (meridional) load, external pressure (circumferential compression) and shear (torsion), respectively. The buckling reduction factors χ_x, χ_θ and χ_t are expressed as a function of the relative slenderness of the shell λ by means of the relationship:

$$\chi = \alpha\chi_{perf} \tag{8}$$

in which α is the imperfection reduction parameter, depending on both the load case and shell slenderness, and χ_{perf} is the buckling factor for a perfect shell, given by:

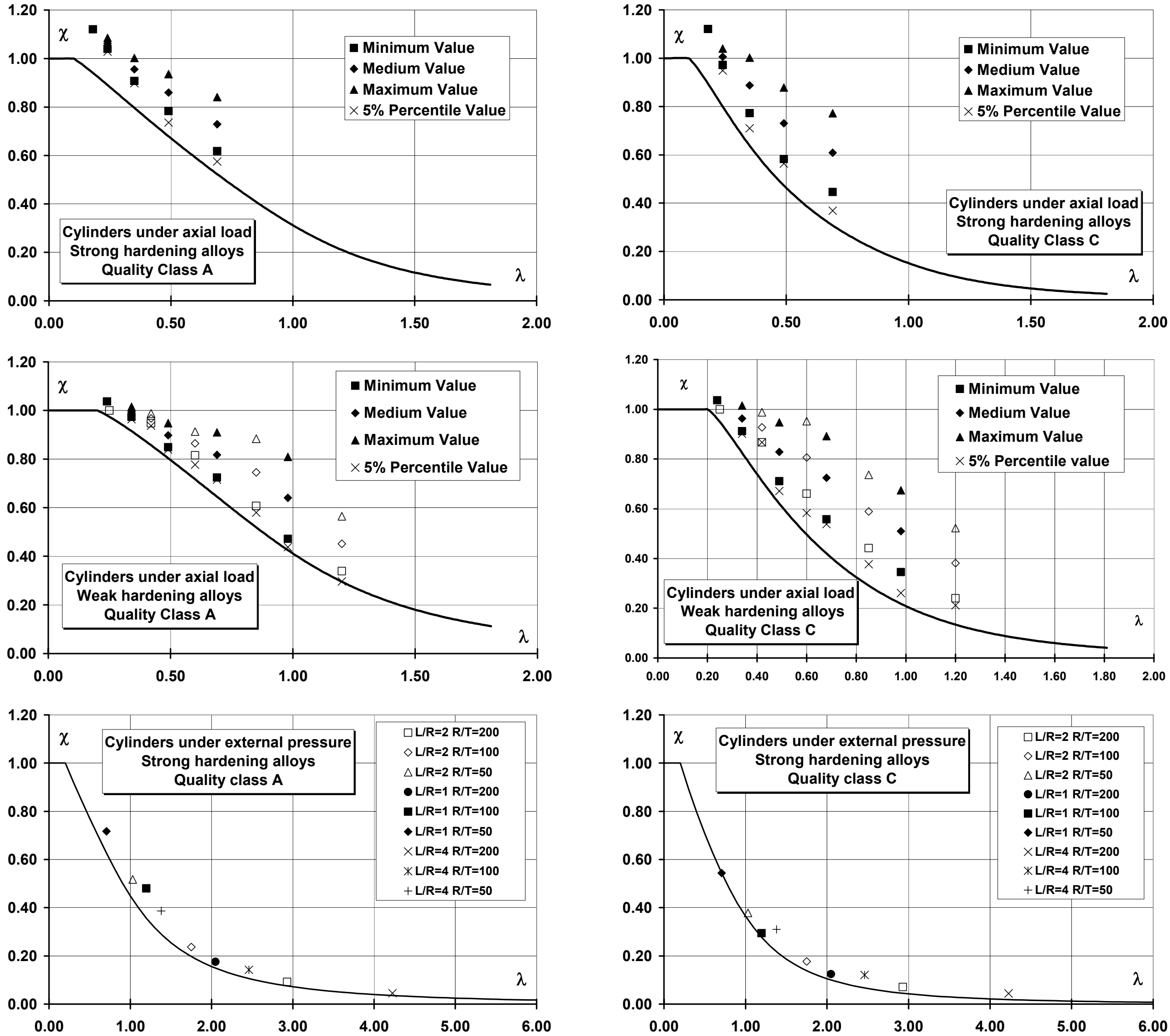
$$\chi_{perf} = 1\Big/\left(\phi+\sqrt{\phi^2-\lambda^2}\right) \qquad \text{with} \qquad \phi = 0.5\left[1+\alpha_0\left(\lambda-\lambda_0\right)+\lambda^2\right] \tag{9}$$

The parameter α_0 depends on the alloy and λ_0 is the squash limit relative slenderness. The relative shell slenderness parameters for the three stress components under consideration are given by:

$$\lambda_x = (f_{0,k}/\sigma_{xRc})^{1/2}, \qquad \lambda_\theta = (f_{0,k}/\sigma_{\theta Rc})^{1/2}, \qquad \lambda_\tau = ((f_{0,k}/\sqrt{3})/\tau_{Rc})^{1/2} \tag{10}$$

Values of α_0 and λ_0 are given in Table 2 for relevant load cases and types of alloy. Equations (8) and (9) are formally identical to those set out in EN1999-1-1 for aluminium members in compression, apart for the value of the shell imperfection reduction factor α, which is given separately for the load cases of axial (meridional) load (α_x), external pressure (circumferential compression) (α_θ) and shear (torsion) (α_τ). In particular, for both external pressure (circumferential compression) and shear (torsion) the same imperfection reduction factors α_θ and α_τ given in ENV1993-1-6 have been kept. A slightly different expression of the imperfection reduction factor α_x has been adopted in the case of axially loaded cylinders, only. This was due to the different imperfection sensitivity exhibited by aluminium cylinders when they buckle in plastic range. Also, the same distinction in quality classes as in ENV1993-1-6 has been kept, which results in keeping the same values of both imperfection limits and quality parameter Q in case of axially loaded cylinders. Summarising, the expressions of α are given in Table 3. Some of the corresponding buckling curves are shown in Figure 5, compared with simulation data. The 5% *Weibull* lower bound has been considered for fitting the curves of axially loaded cylinders.

Note that, contrary to the rule given in ENV1993-1-6, where the same buckling curve and imperfection reduction factors are adopted for external pressure and shear (torsion), different values of λ_0 and α_0 have been adopted in this proposal, in order to have a better fitting of F.E.M. results (see Table 2). This was a consequence of higher buckling factors resulting from numerical analysis for cylinders in shear.



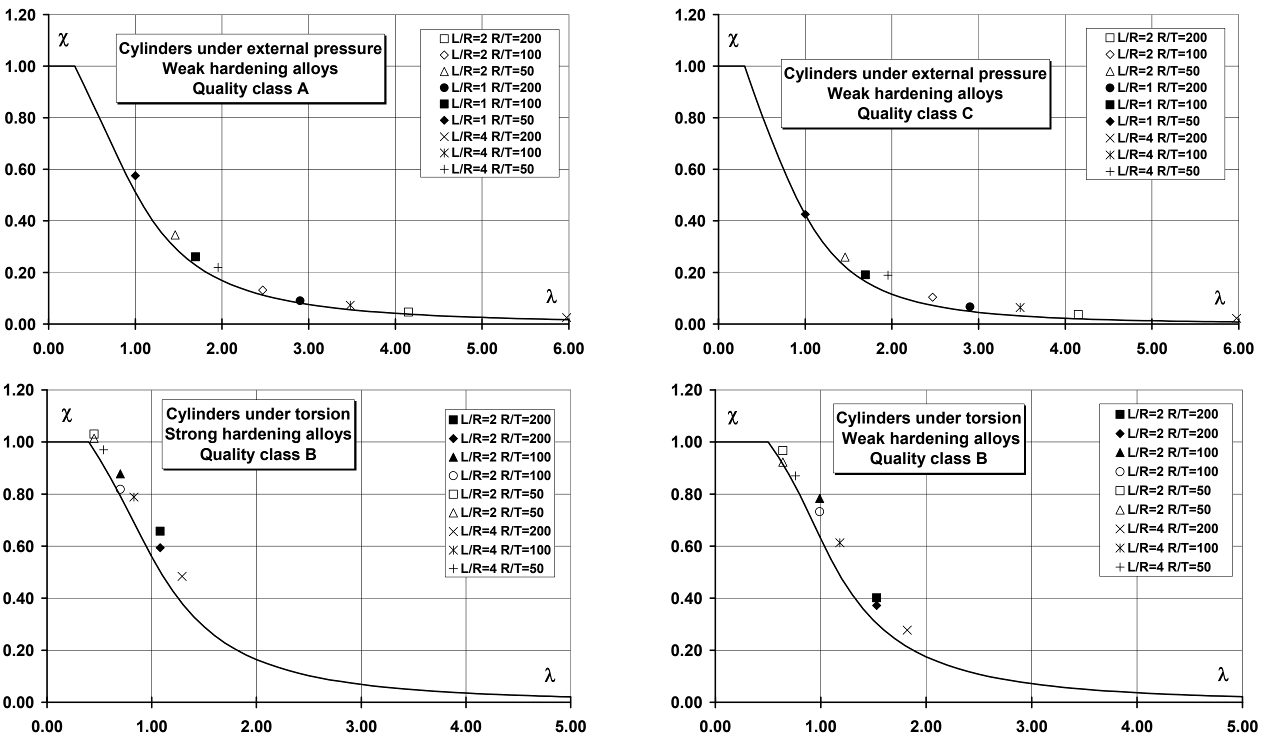


Figure 5 : Comparison of proposed buckling curves with F.E.M. simulation results

CONCLUSIVE REMARKS

The study presented in this paper summarises a proposal for aluminium shell buckling curves discussed within the activity of CEN/TC250-SC9 PT Committee, aimed at issuing the new part Part 1-5 "Supplementary rules for shell structures" of EC9. Curves have been fitted against the results of a wide parametric F.E.M. carried out by means of the non linear code ABAQUS. Load cases referring to axial load, external pressure and torsion (shear) have been addressed, that is the same considered in ENV1993-1-6, dealing with steel shells. The analysis led to a thorough understanding of shell stability, with special emphasis to the case when relevant inelastic deformations occur before buckling. In particular, the interaction between the effect of geometrical imperfections and that of material inelastic behaviour has been underlined.
As EC3 buckling curves have been found inadequate to aluminium shells, the new set of buckling curves discussed herein has been proposed, assuming the basic formulation already implemented in EN1999-1-1 for aluminium members in compression. The obtained curves exhibit a much better fitting of numerical results, with a closer interpretation of results falling in the intermediate slenderness range. They also clearly highlight the difference between weak and strong hardening alloys. For such reasons, the developed curves can be proposed with good conscience for the implementation into EN1999-1-5.
At the issuing date of this paper, October 2003, the proposal is under examination within PT1 of CEN/TC250-SC9. In order to achieve a satisfying degree of homogeneity between formulations for steel and aluminum, a feedback of such discussion is also forwarded to PT1-6 of CEN/TC250-SC3, devoted to the development of Part 1-6 "Supplementary rules for shell structures" of EC3.

REFERENCES

ABAQUS User's Manual, 6.2 (2001), Pawtucket, Rhode Island, Hibbitt, Karlsson & Sorensen, Inc.
Arbocz J., Hol J.M.A.M., 1991, Collapse of Axially Compressed Cylindrical Shells with Random Imperfections, *AIAA Journal*, **29**.
Gerard G. (1962), Plastic Stability Theory of Orthotropic Plates and Cylindrical Shells, Jou. of Aeron. Sci.

Mandara A., Mazzolani F.M. (1993), Stocky Cylinders in Compression: Postcritical Path Evaluation and Collapse Load Prediction with ABAQUS, Proc. of the International ABAQUS Users' Conference, Aachen.
Mazzolani F.M. (1995), *Aluminium Alloy Structures*, 2nd Edition, Chapman & Hall, London.
Mazzolani F.M., Mandara A. (2003), Stability of Aluminium Alloy Cylinders: Report of F.E.M. Analysis and Proposal of Buckling Curves for European Codification. Report of CEN/TC250 SC9 Committee, PT1-1, First Draft, Munich, Second draft, Naples.
Mazzolani F.M., Mandara A. and Di Lauro (2003), Imperfection Sensitivity Analysis of Aluminium Cylinders, III Settimana delle Costruzioni in Acciaio, Genova, Italy.
Mazzolani F.M., Mandara A. and Di Lauro (2004), Remarks on the Use of EC3 Buckling Curves for Aluminium Shells, Proc. of the 10th Nordic Steel Construction Conference, Copenhagen, Denmark.
Mendera Z. (1989), A Uniform Formula of Stability for Cylindrical and Spherical Shells with Imperfections, Proc. of IASS Symp. 10 Years of Progress in Shell and Spatial Structures, Madrid, Spain.
Rotter, J.M. (1998), Shell structures: the new European standard and current research needs, *Thin Walled Structures*, **31**.
Schmidt H. (2000), Stability of steel shell structures: General Report, *Journal of Constructional Steel Research*, **55**.

ACKNOWLEDGEMENTS

This research has been developed within the activity of CEN/TC250-SC9 Committee, chaired by F.M. Mazzolani and devoted to the preparation of Eurocode 9 "Design of Aluminium Structures". The authors gratefully acknowledge the contribution of European Aluminium Association, represented by Mr. J. Luthiger, in the support of mobility expenses of Italian experts.