

# Torsional seismic response of an asymmetric-plan hospital building.

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Keywords: Seismic response, Irregular buildings, Inelastic torsion.

#### ABSTRACT

The inelastic torsional response of an asymmetric-plan hospital building is studied. The response of the structure in the time domain was recorded by highly sensitive sensor network, integrated by a data acquisition system. The identification was performed using techniques of modal extraction in the frequency domain (frequency domain decomposition). A calibration process was applied in order to identify a reliable structural model to be used for the seismic vulnerability assessment of the hospital building. In particular, a nonlinear static procedure accounting for mass distribution, higher modes contribution and mode-shapes correlation was proposed for the estimation of the seismic response of irregular buildings. Finally, the influence of lateral force distribution, node control during pushover and accidental eccentricity is investigated.

## 1 INTRODUCTION

Torsion in buildings during earthquake ground motions is generated not only by non-symmetric distributions of mass and stiffness, but also due to other causes difficult to predict and quantify that may occur generating additional eccentricities. such as excitation differences at the support points, stiffness and strength of non-structural elements, non-symmetric distributions of live loads. The torsional response may be intensified the inelastic range due to increased in eccentricities caused by yielding in the perimeter of the structure and by torsional coupling effects especially under bidirectional seismic excitation. generally decrease Torsional effects with increasing intensity of ground motion and with increase related of plastic deformations. However, the linear analysis may be not conservative, especially for the stiff edge in the strong direction of torsionally stiff buildings and for the stiff edge in the weak direction of torsionally flexible buildings. The application of nonlinear static procedures to multistorev irregular buildings requires various problems to be solved: 1) direction of seismic excitation; 2) eccentricity of lateral force distribution; 3) higher modes contribution; 4) node control for monitoring the target displacement. For these structures the conventional pushover analysis with lateral force applied in the centre of mass of the building may underestimate the seismic torsional response obtained from step-by-step time-history analysis. Furthermore, the use of the centre of mass as node control may influence the accuracy of nonlinear static procedures based on the capacity spectrum method.

#### 2 CASE STUDY: ASYMMETRIC -PLAN HOSPITAL BUILDING

The hospital is composed of RC wall-frame buildings designed and constructed in 70's. The building is composed of different structures separated by seismic joints (Figures 1,2). The study is carried out on an irregular T" plan shape building designed for earthquake action of Italian old seismic code (N.1684 November 25th, 1962). Both destructive and non-destructive testing methods were applied for the building diagnosisstate.



Figure 1. Aerial view of the Hospital building of Avezzano.



Figure 2. Plan view of the Hospital of Avezzano.

In particular, 53 monotonic compressive tests on cylindrical specimens, 24 tensile tests on steel rebars, ultrasonic tests, 40 carbonation depth measurement test, 163 Schmidt rebound hammer tests, 180 radiographic tests. The compressive strength was finally estimated by the combined Sonreb method. The mean value of the compressive strength of concrete on cylindrical specimens is  $f_{cm}=234 \text{ daN/cm}^2$ ; the mean value of steel strength tensile of rebars  $f_{ym}$ =4026daN/cm<sup>2</sup>. Geological and geotechnical tests were carried out to evaluate the soil profile and to determine the ground type according to Eurocode 8 and new Italian Code (2008). In particular, the following in-situ tests were performed: N.1 soil profile test, N.3 Standard Penetration Tests (SPT) and N.1 Down-hole Test which determines soil stiffness properties by analysing direct compression and shear waves along a borehole down to about 30m (Tab 1). The results obtained give the following classification: ground type C ( $180 \le V_{S,30} \le 360$  and  $15 \le N_{SPT} \le 50$ ). The building is erected on a flat ground (topographic amplification factor  $S_T=1.00$ ).

Table 1. Soil profile.

Ν	Layer	Thickness	Depth	
[		(m)	(m)	
1	Organic soil	1.50	0.00	
2	Sand gravel soil and pebbles	13.50	1.50	
3	Clay	2.00	15.0	
4	Silt with clay	5.00	17.0	
5	Silt with gravel	8.00	22.0	

#### 3 ENVIRONMENTAL VIBRATION TEST AND STRUCTURAL IDENTIFICATION

The forced vibrations applied, for instance, by a mechanical vibrodyne, are not suitable to be used on full functional hospital structures, because even small vibration cannot be tolerated in such conditions. On the contrary, measurement of environmental vibrations may be carried out without direct excitation on the building using the natural noise and vibration such as wind and traffic loads. An experimental study was developed to evaluate the dynamic behaviour of the structure and the dynamic interaction between the natural frequencies of the building and the excitation forces. The in-situ experimental tests were performed applying the environmental vibration testing method. This method is a relatively simple, requires equipment easy to be transported and such a test can be conducted even if the structure is in use. This aspect is essential especially in the case of strategic structures (emergency management centres, hospitals and so on) whose function cannot be interrupted.



Figure 3. Equipments used for environmental vibration test.

The ambient vibration measurement allowed, after a careful choice of the positioning of the sensors, to get natural frequencies and vibration modes from the direct measurement. The response of the structure in time domain was recorded by highly sensitive sensors, accommodated with a data acquisition system. The instrumentation used included (Figure 3): N.16 PCB piezoelectric accelerometers (Piezotronics model 393B04); N.1 data acquisition (National board Instruments DAQCard-16XE50); connector block for interfacing I/O (input/output) signals to plug-in data acquisition device. The accelerometers were appropriately calibrated following the manufacturers' suggested procedures. The environmental vibration testing under wind and traffic vibration was monitored on July 2008.

Table 2. Location of equipment during vibration tests.

Ν	Accel.	Dir	Condit.	Height	Floor	Column
1	А	х	C1	4.11	1	1x
2	В	у	C1	4.11	1	1 y
3	С	х	C2	7.73	2	1x
4	D	х	C2	7.73	2	1y
5	E	у	C2	11.35	3	1y
6	F	у	C3	4.11	1	3y
7	G	у	C3	4.11	2	3y
8	Н	У	C3	11.35	3	3y
9	Ι	у	C4	4.11	1	2y
10	L	у	C4	7.73	2	2y
11	М	У	C4	14.97	4	2y
12	Ν	y	C5	14.97	1	1y
13	0	x	C5	18.57	5	1x
14	Р	у	C5	18.57	5	1y
15	Q	y	C6	14.97	4	3y
16	R	у	C6	18.57	5	3y



Figure 4. Location of the sensors during environmental vibration tests.

The location of the accelerometers and the conditioners, and the vertical "columns" for the calculation of the mode shapes are reported in Figure 4 and in Table 2. The location of the devices during vibration testing was selected from the preliminary model (Figure 5). The spectral analysis of the recorded signals may give the natural frequencies and the corresponding mode shape. Usually, the signal recorded with this technique is very low as well as the signal-tonoise ratio. This means that the recorded signal must be amplified and processed, and the frequencies negligible be filtered (local and partial vibration and phenomenon of the signal transferring with frequencies in the range 0.50-20Hz). The data acquisition was realized in Labview 8.0 with sampling frequency of 100 Hz. The Fast Fourier Transform (FFT) was used to determine the frequency spectrum of the signal processed through a 30 Hz low-pass filter. The experimental and theoretical procedure starts from an assumption that the exciting forces are a

1° FLEXURAL Y - TORSIONAL



1° FLEXURAL X - TORSIONAL



Figure 5. Mode shapes of the hospital building model

stationary stochastic process with a relatively flat frequency spectrum. The identification was performed using techniques of modal extraction in the frequency domain. These techniques allow the assessment of natural frequencies, modal damping and mode shapes. The Fast Fourier Transform (FFT) was used to determine the frequency spectrum of the signal processed through a 30 Hz low-pass filter. An often more useful alternative is the power spectral density (PSD), which describes how the power of the signal is distributed with frequency.

Table 3. Natural frequencies from on-site monitoring and numerical modelling.

Description	Frequency From test (Hz)	Frequency From model (Hz)
1° Flexural Y – Torsional	2.53	2.59
Torsional	3.54	2.87
1° Flexural X	3.67	3.52
2° Flexural Y – Torsional	7.95	8.19
2° Flexural X	9.06	11.1



2° FLEXURAL Y - TORSIONAL







Figure 7. Lateral displacement patterns

In Figure 6 the resonant frequencies are identified and located at the evident peaks of PSD spectrum. The use of these experimental data with the analytical model allows for a verification of the adequacy of the model and for its calibration. At first, a preliminary model was developed for selecting the location of the sensors during vibration testing. In particular, a detailed numerical model of wall-frame building was implemented in SAP 2000 computer program. During the calibration process the values initially adopted were successively corrected in order to identify a reliable structural model to be used to have an accurate seismic vulnerability assessment of the hospital building. In particular in the refined model the following aspects are considered: 1) modelling of non-structural infill panels with the well-known equivalent diagonal strut model (cross section 40x60; Young's modulus  $E=5350 da N/cm^2$ ; 2) modelling of floors as orthotropic shells rather than constraints diaphragms; 3) calibration of Young's modulus of concrete; 4) inclusion of stiffening RC members that are present at the first, third and sixth floor; 5) calibration of live loads; 6) use of rigid end offsets for the beam elements. In Table 3 the natural frequencies obtained from the calibrated model are compared with the frequencies derived from the environmental vibration test. A good correlation is found especially for 1st and 2nd flexural Y-torsional mode shape and for 1st flexural X mode shape. Finally, in Figure 7 the comparison between experimental and numerical mode shapes in terms of displacement pattern is carried out. The results are referred to the locations COL 1X and COL 2Y of the sensors (Figure 4) that are close to the centre of stiffness of the building. A very good agreement between experimental and numerical patterns is observed. However, it must be noticed that this results is obtained only for the sensors that are close to the centre of stiffness of the building (COL 1X, COL 2Y). On the contrary, for the other accelerometers the torsional effects and the higher modes contribution makes more difficult to extract the peaks from the PSD function because there are a great number of very close peaks. In this case the lateral displacement pattern of the mode shapes from numerical model is very sensitive to the value of frequency, and it may be strongly different from the experimental displacement pattern.

# 4 INELASTIC TORSIONAL RESPONSE

# 4.1 Behaviour of asymmetric-plan buildings

In order to deal with torsional effects modern codes have introduced the so-called accidental design eccentricity to be used to displace the mass in every floor also in the case of fully symmetric buildings. This provision is based on the studies about torsional response of buildings that are carried primarily using simplified elastic multi-storey buildings or simplified inelastic, one-story systems, while general conclusions regarding the inelastic torsional response of real multi-storey building are still lacking. Many studies focused on the identification of the most significant parameters governing the nonlinear behaviour of asymmetric-plan buildings: the eccentricity between the centre of stiffness and the centre of strength (Chopra et al. 2004), the inplane asymmetry distribution, the bidirectionality of the seismic excitation, the uncoupled translational and rotational frequencies ratio (Fajfar et al. 2005), the ground motion properties in frequency, intensity and duration. Nonlinear dynamic analyses of asymmetric building structures have also been performed in connection to the development of the 3D pushover analyses (Kilar et al. 2001). Stathopoulos et al. (2005) studied the problem of inelastic torsion by means of multi-storey inelastic building models. In recent years, Lucchini et al. (2008) presented results from nonlinear dynamic analysis on single-storey frame buildings characterized by different strengths distributions and excited by ground motions of increasing intensities. Bosco et al. (2008) proposed a procedure based on two nonlinear static analyses with two different corrective eccentricities determined analysing statistically the response of a wide set of idealized one-storey systems. De Stefano et al. (2008) found that the envelope of lateral displacements at the top floor obtained with elastic dynamic analysis is generally conservative for frame structures.

## 4.2 Pushover for torsionally irregular buildings

The validity and applicability of the static pushover analysis have been extensively studied in literature, and implemented in procedures based on Capacity Spectrum Method (CSM) or

Displacement Coefficient Method (DCM), such as in FEMA 273, FEMA 356 (2000), ATC-40 (1996), Eurocode 8 (2004), Italian Code (2008), FEMA-440 (ATC-55, 2003), ASCE/SEI 41-06 standard (ASCE, 2007). The application of pushover analysis to real multistorey buildings may create some problems connected to their irregularity in plan and/or in elevation. In fact, although the formulation for inelastic response of asymmetric building under earthquake motions was extensively studied in 70s, only in recent years procedures have been proposed to extend pushover analysis to asymmetric-plan the buildings. In fact, the pushover nonlinear analysis of plan-asymmetric buildings proved to be a very difficult problem. Some authors observed that the torsional effects generally decrease with increasing intensity of ground motion and with increase of plastic deformations. related Consequently, а conservative estimate of torsional effects may be determined by the results of elastic modal analysis and the global displacement demand may be determined by unidirectional pushover analysis of 3D structural model. However, the torsional response in the elastic and inelastic range is not similar for the stiff edge in the strong direction of torsionally stiff buildings and for the stiff edge in the weak direction of torsionally flexible buildings. Some authors have observed that while the first mode contribution requires a nonlinear analysis to be determined, the response of higher modes may be estimated by linear analysis. Consequently, they proposed to calculate the torsional response by the combination of the inelastic first mode contribution with the elastic higher mode contributions. In particular, in the Modified Modal Procedure Analysis (Chopra et al. 2004) the first mode contribution is determined by nonlinear static analysis using two lateral forces and torque at each floor level for each mode. The higher mode effects on seismic demand are calculated from linear elastic analysis and then combined using the COC rule in order to obtain an estimate of the total inelastic demand of the building. The extension of N2 method (Fajfar 2005) is based on conventional pushover analysis of a 3D model of the building using a modal horizontal load pattern with a target displacement computed from inelastic demand spectra. Torsional effects are considered by amplifying pushover analysis results by an amplification factor, determined from elastic modal analysis of the 3D building as the ratio of horizontal nodal displacement to the corresponding displacement at the mass centre of the level considered. A new procedure, called Force/Torque Pushover (FTP) analysis, to select storey force distributions for 3D pushover analysis of plan-irregular RC frame structures was proposed by Ferracuti et al. (2009). The force distribution is proportional to the fundamental mode shape. The floor force resultant is divided into lateral forces in X- and Y-directions and a torque with respect to the centre of mass. A weight coefficient for the two components (Force/torque) has to be calibrated to capture the more severe configurations depending on the degree of irregularity of the structure. These pushover methods tend to have some problems to give consistently good agreement with the Response History Analysis (RHA) results for both the stiff and the flexible sides. In general, the agreement is better at the centre of mass while deteriorates at the two edges where the torsional motion amplifies or de-amplifies the translational response. Moreover, the differences tend to increase as the motion intensity increases and the response becomes more nonlinear.

## 4.3 Proposed procedure: 3D CQC load pattern

In this paper, the torsional effects are evaluated by a CQC distribution of the lateral loads. In particular, the load  $F_k(x,y)$  to be applied at the kth floor in the node of coordinates (x,y) is proportional to the mass  $m_k(x,y)$  associated to the node and to the displacement  $U_k(x,y)$  defined by the CQC combination of the modal lateral displacements calculated from the response spectrum analysis of the building, including sufficient modes to capture at least 90% of the total mass, as follows (3D CQC Distribution):

$$U_k(x,y) = \sqrt{\frac{1}{\omega_i^2 \cdot \omega_j^2}} \phi_{ik}(x,y) \phi_{jk}(x,y) G_i G_j S_a(T_i) S_a(T_j) \rho_{ij}}$$
(1)

where  $\phi_{ik}(x,y)$  is the shape of the ith mode at the kth floor;  $G_j$  is the corresponding participation factor;  $S_a(T_i)$  is the spectral acceleration,  $\rho_{ij}$  is the correlation coefficient between the mode shapes.

#### 4.4 Estimation of inelastic response

The inelastic torsional response of the wall-frame hospital building was evaluated with a model implemented in SAP 2000 computer program (2010). In particular, a Coupled PMM hinge model for the members of the framed structures

and a beam-column element model for the RC walls were considered in the analysis. The coupled PMM model has some computationally advantages over distributed plasticity models, but it may suffers some limitations to capture the member behaviour under the combined actions of compression, bi-axial bending and buckling effects, which may significantly reduce the loadcarrying capacity of the structure. The length of plastic hinge was calculated with the Italian Code formula (2008). The beam-column joint is represented as a rigid zone having horizontal dimensions equal to the column cross-sectional dimensions and vertical dimension equal to the beam depth. A fiber element uniaxial model for confined concrete is used. In particular, the concrete stress-strain model is an enhanced version of the well-known model of Mander. et al. (1988). Steel was modeled with an elasticplastic-hardening relationship. A one-component beam-column element model is adopted for predicting the inelastic response of RC structural walls. This model consists of an elastic flexural element with a nonlinear rotational spring at each end to account for the inelastic behavior of critical regions. The fixed-end rotation at any connection interface can be taken into account by a further nonlinear rotational spring. The non linear static analyses were carried out considering parameters: the following 1) accidental eccentricity of lateral force distribution; 2) node control for monitoring the target displacement; 3) lateral force distribution. In particular, an accidental eccentricity of the storey mass equal to  $\pm 5\%$  of planar dimension orthogonal to the direction of earthquake ground motion is considered. Three different locations for the node control are used for monitoring top-floor target displacement: A) stiff edge; B) center of mass; C) flexible edge. Finally, the 3D CQC distribution here proposed is compared to Equivalent Lateral

Force Distribution (ELFD) and Uniform Distribution (UD).

In Figure 8 the capacity curves (Base shear vs displacement) floor and the points top corresponding to the limit states are reported. The seismic vulnerability assessment was carried out with the four performance levels considered in Italian seismic code (2008): Operational Limit State (SLO), Damage Limit State (SLD), Life Safety Limit State (SLV), Collapse Prevention Limit State (SLC). The parameters of the elastic demand response spectra are synthesized in Table 4. In Table 5 are reported the risk indices defined by the capacity/demand quotients in terms of peak ground acceleration. In particular, both ductile and brittle (shear failure of structural elements, failure of beam-column joints) failure modes are considered in the analysis. The results obtained show that both the ELFD and the UD distributions may overestimate the risk index when compared to CQC distribution, and so they are conservative for the vulnerability assessment.

Table 4. Parameters of elastic response spectra (NTC8)

Parameter	SLO	SLD	SLV	SLC
Prob.of excedence P <sub>VR</sub>	0.81	0.63	0.10	0.05
Return Period $T_R$ (years)	120	201	1898	2475
Peak ground accel. ag/g	0.144	0.178	0.397	0.433
Amplification factor F <sub>o</sub>	2.297	2.315	2.425	2.434
Transition Period $T_{C}$ (s)	0.471	0.485	0.537	0.542

Table 5. Risk indices

Force	Node	Brittle	Ductile Failure Modes			
Distr.	Control	Failure	$\alpha_{SLO}$	$\alpha_{SLD}$	$\alpha_{SLV}$	$\alpha_{SLC}$
ELFD	А	0.10	1.16	1.20	1.00	1.22
ELFD	В	0.10	1.13	1.14	0.91	1.13
ELFD	С	0.10	0.92	1.00	0.81	1.08
UD	А	0.09	1.41	1.48	1.31	1.36
UD	В	0.09	1.16	1.24	1.11	1.25
UD	С	0.09	1.06	1.17	1.03	1.20
CQC	А	0.10	0.86	0.85	0.72	0.80
CQC	В	0.10	0.95	1.00	1.00	1.17
CQC	С	0.10	0.90	1.00	1.00	1.15



Figure 8. Variation of capacity curve with lateral force distribution.



Figure 11. Pattern of lateral displacement: 1) ELFD Distribution; 2) UD distribution; 3) 3D CQC distribution.

In Figure 9 the variation of capacity curve with node control is shown. The results obtained give evidence of the sensitivity of the capacity curve to the node control, especially for e/L=0.05 when the accidental eccentricity has the same sign of the structural eccentricity (defined as the offset of the centre of stiffness CS from the centre of mass, CM). In Figure 10 the variation of capacity curve with the accidental eccentricity (e/L=-0.05); e/L=+0.05; e/L=0) is reported. The agreement is better at the centre of mass while deteriorates at the two edges where the torsional motion amplifies or de-amplifies the translational response. However, it seems evident that the capacity curve is not much sensitive to the accidental eccentricity. On the contrary, the pattern of lateral displacement is influenced both by the accidental eccentricity and by the lateral force distribution (Figure 11). In particular, for e/L=0.05 the addition of accidental eccentricity to structural eccentricity strongly increases the torsional rotation. In this case, both the ELFD and the UD distributions understimate this effect when compared to COC distribution, and so they may be inaccurate for the estimation of torsional inelastic response.

## 5 CONCLUSIONS

The irregular T" plan shape of the building makes the modal identification from environmental vibration test very sensitive to the location in plan of the accelerometers. In particular, a very good agreement of experimental and numerical modal proprieties was found only for the sensors located very close to the centre of mass of the building. On the contrary, for the signals recorded very far from the centre of stiffness the torsional effects and the higher modes contribution generally produce very close peaks. The results of fnonlinear static analyses show that both the equivalent lateral force distribution and the uniform distribution may overestimate the capacity of the structure, particularly on the flexible edge. The effectiveness of the procedure may be improved combining the effects obtained under the multimodal distribution that give the maximum displacement, with the results of another pushover analysis under the multimodal distribution that give the maximum rotation. In other words, the most severe conditions may be obtained with two pushover analyses.

### ACKNOWLEDGEMENTS

The author wishes to thanks Mr. S. Della Volpe, Mr. N. Nappa, and the professional team belonging to Studio KR & Associati, Lenzi Consultant and Aires Ingegneria. Many thanks also to ASL (Hospitals complex) No.1 of Avezzano-Sulmona represented by Mr. F. Dalla Montà (P.E.) for the support during testing.

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